\|| **PG – 383**

I Semester M.Sc. Degree Examination, January 2017
(CBCS)
MATHEMATICS
M 103T : Topology – I

Time : 3 Hours   Max. Marks : 70

Instructions : *i) Answer* ***any five full*** *questions.*
ii) ***All*** *questions carry* ***equal*** *marks.*

1. a) What do you mean by denumerable set ? Is Q, the set of rational numbers denumerable ? Justify. **4**

   b) Show that :

   i) Superset of an infinite set is infinite.

   ii) Subset of a finite set is finite. **6**

   c) Does every infinite set contains a denumerable ? If yes, explain. **4**

2. a) State Schroder-Bernstein theorem.

   Use it to prove that $(0, 1) \sim [0, 1]$. **3**

   b) Let C = card R. Show that C.C = C. **6**

   c) If P(A) denote the power set of a set A then prove that card A < card P(A). **5**

3. a) Let (X, d) be a metric space and $d(x, y) = \dfrac{d(x, y)}{1 + d(x, y)}$.

   Show that (X, d_1) is a metric space by checking only triangle inequality for d_1. **3**

   b) Show that if a convergent sequence in a metric space has infinitely many distinct points then its limit is a limit point of the set of elements of the sequence. **4**

   c) Prove that a subspace Y of a complete metric space is complete if it is closed. Is the converse true ? Explain. **7**

P.T.O.

4. a) Prove that if a metric space X is complete then for every nested sequence $\{F_n\}_1^\infty$ of a nonempty closed sets in X with $\delta(F_n) \to 0$, $\bigcap_{n=1}^{\infty} F_n$ is a singleton set. **6**

   b) Define a set of first category. State and prove Baire's category theorem. **8**

5. a) Prove contraction mapping theorem. **6**

   b) Show that every metric space has a completion. **8**

6. a) Show that intersection of two neighborhoods is a neighborhood. Is superset of a neighborhood is again a neighborhood ? Justify. **4**

   b) Show that :

      i) an arbitrary intersection of closed sets is closed.

      ii) a set containing all its limit points is closed. **6**

   c) Show that the interior of intersection of two sets is the intersection of their interiors, but this is not the case for the union of two sets. **4**

7. a) Let (X, τ) be a topological space. Show that a subfamily $\mathscr{B}$ of τ is a base for τ if and only if for every $U \in \tau$ and $x \in U$ there is a $B \in \mathscr{B}$ such that $x \in B \subseteq U$. **6**

   b) Show that a function $f : X \to Y$ is continuous if and only if inverse of every open set in Y is open in X. **4**

   c) Show that a bijective function $f : X \to Y$ is a homeomorphism if and only if $f(\overline{A}) = \overline{f(A)}$, for all $A \subseteq X$. **4**

8. a) If C is a connected subset of (X, Y) which has a separation $X = A \cup B$ then prove that either $C \subseteq A$ or $C \subseteq B$. **4**

   b) Show that closure of a connected set is connected. **5**

   c) Prove that union of family of connected sets with non-empty intersection. **5**
